Geometrically-Accurate Topology Simplification of Triangulated Cortical Surfaces Using Non-Separating Loops

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Simplification de la Topologie des Surfaces Corticales

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Abstract

In this paper, we focus on the retrospective topology correction of surfaces. We propose a technique to accurately correct the spherical topology of cortical surfaces. Specifically, we construct a mapping from the original surface onto the sphere to detect topological defects as minimal non-homeomorphic regions. The topology of each defect is then corrected by opening and sealing the surface along a set of non-separating loops that are selected in a Bayesian framework. The proposed method is a wholly self-contained topology correction algorithm, which determines geometrically-accurate, topologically-correct solutions based on the MRI intensity profile and the expected local curvature. Applied to real data, our method provides topological corrections similar to those made by a trained operator.
Résumé

Nous proposons une méthode pour la correction de la topology de surfaces triangulées. La topologie d’une région non-homéomorphe à celle d’un disque est corrigée de manière optimale par rapport à l’information disponible (e.g. courbure, intensité, …). L’espace des corrections topologiques est exploré aléatoirement et la solution correspondante au maximum-a-posteriori est sélectionnée dans une approche Bayesienne. Appliquée au problème de la reconstruction des surfaces corticales à partir d’image IRM, notre méthode produit des surfaces corticales avec une géométrie précise et une topologie exacte ; les solutions effectuées par notre algorithme sont similaires à celles proposées par un opérateur entrainé.
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1 The Cortical reconstruction problem

The human cerebral cortex is a highly-folded ribbon of gray matter (GM) that lies inside the cerebrospinal fluid (CSF) and outside the white matter (WM) of the brain. Locally, its intrinsic “unfolded” structure constitutes a two-dimensional (2-D) sheet, which is several millimeters thick. In the absence of pathology and assuming that the midline hemispheric connections are artificially closed, each cortical hemisphere can be viewed as a simply-connected 2D sheet of neurons that carries the simple topology of a sphere (Fig. 1).

Recently, there has been a research focus on the extraction of accurate and topologically-correct models of the brain surface. The development of these algorithms greatly facilitates the analysis of cortical data [1, 2], and alleviates many problems of the three-dimensional embedding space (such as the underestimation of true cortical distances or the overestimation of cortical thicknesses). Certain clinical and research applications depend crucially on the accuracy and correctness of the representations: visualization [1, 2, 5], spherical coordinate system and surface-based atlases [1, 2, 4, 5, 6, 7, 8], shape analysis [8, 9, 10, 11, 12, 13], surface-based processing of functional data [1], and inter-subject registration [5, 14, 15], among others.

Many recent segmentation algorithms for neuroimaging data are able to identify and precisely locate diverse brain structures, although typically without ensuring the validity of the final topology (i.e. that of a sphere). Magnetic Resonance Image (MRI) often contain various artifacts (e.g. image noise, image intensity inhomogeneity or non-uniformity, caused by RF inhomogeneities, partial volume averaging effects, and subject motion) that are difficult to predict and model. In the case of cortical segmentations, partial volume effects makes the accurate location of the surface of the cortex particularly difficult (Fig. 1). Because of its highly-folded nature, opposite banks of a sulcus often appear connected, and small gaps between adjacent folds of the neocortical gray matter become invisible in the finite resolution MR images (Fig. 1b, c). Conversely, thin strips of white matter often appear darker due to neighboring gray matter [5], and the interface between GM and WM is often pierced by incorrectly identified GM in many segmentations (Fig. 1d). These topological defects in the segmentation are hard to detect and correct retrospectively, with the automatic extraction of accurate and topologically-correct cortical surfaces remaining a challenge.

\[3\] In mathematical terms, two surfaces have the same topology if they are homeomorphic.
\[4\] The true topology of the gray/white surface is not one of a sphere, as a result of the midline connections between the left and right hemisphere, such as the anterior and the posterior comissures.
\[5\] Particularly in the case in the parahippocampal gyrus, where the white matter is only a voxel or two thick.
Figure 1: a) The human cerebral cortex is a highly-folded ribbon of gray matter that lies inside the cerebrospinal fluid build and outside the white matter of the brain. The green surface represents the interface between WM and GM, and the red surface (i.e. the pial surface) models the interface between GM and CSF. When the midline connections between the left and right hemisphere are artificially closed, these two surfaces have the topology of a sphere. b) 3D rendering of the highly-folded pial surface. Opposite regions of a sulcus are often contiguous. c) As a result of the partial volume effect, it is difficult to distinguish opposite banks of a sulcus from one another. d) Segmentation algorithms that do not constrain the topology often create cortical segmentations with certain topological defects (i.e. handles). Close-up of a topologically-incorrect gray/white surface representation.

2 State of the Art in Segmentation under Topological Constraints

Segmenting under topological constraints is difficult. Segmentation algorithms (see [16], [17], [18] for some good reviews), which operate on the intensity or variations of the texture of the image, are sensitive to the artifacts produced by the image acquisition process. Most often, segmentation techniques that do not integrate any topological constraints generate segmentations that contain small deviations from the true anatomy of the structures of interest. In the case of cortical segmentations, these deviations can form handles (or holes, which are topologically equivalent - Fig. 2a) that erroneously connect parts of the volume.

Integrating topological constraints into the segmentation process significantly complicates the task. Topology is both a global and a local property; small and local modifications of a geometric shape can change its global connectivity. At the same time, topology is intrinsically a continuous concept, and topological notions are difficult to adapt into a discrete framework. For these reasons, the number of techniques available and applicable to the segmentation of images is quite limited.

Methods creating topologically-correct cortical models can be divided into two categories: topologically-constrained segmentation methods that directly incorporate topological constraints into the segmentation process, and retrospective topology correction techniques that aim to correct the topology of already-
segmented images. For a complete review, refer to [19].

2.1 Topologically Constrained Segmentation

Approaches that integrate the topological constraint directly into the segmentation process have the advantage of allowing the user to explicitly specify the topology of the final segmentation. A model, carrying the desired topology, is iteratively deformed onto the cortical surface while preserving its topology. To this end, active contours [20] (explicit representations [3, 21, 22, 23, 24, 25, 26] and implicit representations [27, 28, 29]), digital models [30, 31, 32, 33], and registration methods [34, 35, 36] have proven to be extremely useful. Unfortunately, the energy functionals driving the deformation are highly non-convex, and attaining the desired final surface most often requires an initialization of the model that is close to its final configuration. In addition, local topological constraints can easily cause large geometrical inaccuracies that are difficult to identify and correct.

2.2 Retrospective Topology Correction

Recently, new approaches have been proposed to retrospectively correct the topology of an already segmented image. These techniques, which do not enforce any topological constraints into the segmentation process, focus on more accurate models. Many segmentation techniques, using local intensity, prior probabilities, and/or geometric information, while giving no consideration to topology, generate accurate cortical surfaces with few topological inconsistencies. Retrospective techniques aim to identify and correct these defects in order to produce geometrically-accurate topologically-correct surfaces.

Most retrospective methods do not make full use of all available information. They assume that the topological defects in the segmentation are located at the thinnest parts of the volume and aim to correct the topology by minimizing the number of modifications in the original segmentation [37, 38, 39, 40]. These methods, which rely on the accuracy of the initial segmentations, often produce accurate topological corrections. Yet, the resulting solutions are rarely geometrically-accurate as they do not correspond to the ones that a trained operator would make. Additional information, such as the expected local curvature or the local intensity distribution, may lead to different corrections, i.e. possibly comparable to the ones a trained operator would make. This is illustrated by Fig. 2.

Only a handful of techniques have been proposed to integrate additional information into the topological correction process [19, 41, 42, 43]. Yet, for a given topological defect, these methods fail to generate multiple potential solutions that are requisite for selecting the expected correction. In fact, they produce only two
Methods that aim to correct the topology of a segmentation by minimizing the number of modifications in the segmentation might not produce valid corrections. a) A topological defect is identified in red on the original topologically-incorrect cortical surface. A topological defect can be interpreted as a handle (in green) or, equivalently, as a hole (in brown). b) Cutting the handle: an incorrect topological correction based only on the size of the defect. In this case, cutting the handle corresponds to a “smaller” modification of the surface than filling the corresponding hole. However, this topological correction is not the right one and brings about an inaccurate cortical representation. c) Filling the hole: correct topological correction realized by a trained operator. Note that the correction filling a hole is equivalent to cutting a “background” handle. Another point to note is that the correct modification cannot be determined solely from the surface - the intensity volume is required in order to assess the correct location of the true surface.

candidate solutions. Two potential solutions that correct a handle are cutting the handle or filling the corresponding hole (i.e. cutting the “background” handle). However, the exact location of these potential corrections (i.e. the resulting shape of the potential corrections) is often determined by criteria that ignore the underlying MRI intensity profile and/or the local curvature, which means that the resulting corrections are never optimized relative to these parameters. Other solutions, such as the ones a trained operator makes, do not arise. This problem arises when the proposed framework does not examine multiple candidate solutions. Figure 3 illustrates the difficulty of finding the correct solution when the topological defect is complex.

To our knowledge, only one approach [44] has been proposed to achieve optimal topological corrections integrating a wide range of information available in the image. Similar to the approach developed by Fischl et al. [42], we construct a mapping from the initial triangulation onto the sphere and locate topological defects as locally non-invertible regions. We introduce a genetic algorithm to explore the space of possible surface retessellations and to select an optimal configuration. However, this approach suffers from several drawbacks (described in the next section), and topological corrections are not always the desired ones. We
Figure 3: Some topological defects are complex and extremely hard to correct. Existing methods (with the exception of [44]) only produce a few potential topological solutions, with the chosen solution rarely the expected one. a) A complex topological defect comprised of 3 handles. b) One sagital MRI slice of the topological defect, illustrating the complexity of the underlying MRI intensity profile. c) Proper solution realized by a trained operator. d) Sagital cut of the MRI volume showing the location of the surface of the corrected defect.

then propose several extensions that we believe resolve these issues, yielding an accurate and automated method for generating topologically correct models of the human cerebral cortex.

2.3 Approach

For a given topological defect, the MRI intensity profile contains important information regarding the location and position of the potential topological correction. The corrected surface should be located at the border of the white and gray matter, with white matter inside and gray matter outside the surface. In addition, the smoothness of the corrected defect should be comparable to the smoothness of the remainder of the cortical surface. Incorporating this information into the topology correction procedure can exclude classes of inaccurate solutions, leading to a significantly more robust and accurate technique.

Our method is based on the notion that if a surface has spherical topology it cannot contain any non-contractible curves (i.e. handles). More technically, this can be restated as “every closed 2-manifold that is homeomorphic to a sphere is simply-connected” ([45, 46, 47]). This of course is the two-dimensional analog of the well-known Poincare conjecture (one of the seven Millennium Prize Problems selected by the Clay Mathematics Institute [48]), which appears to have been recently proven by G. Perelman [49, 50, 51].

The approach follows from our previous work [44]. Although the previous method often yields valid solutions, it has some limitations:

- **Spherical mapping:** the space of potential retessellations depends on the initial mapping. In [19], we address this problem by generating an array of distinct mappings. However, the resulting procedure is time-consuming
and not well-adapted to large topological defects, such as the ones that often arise in the medial temporal lobe and temporal poles.

- **Self-intersections:** the retessellation procedure, which does not directly guarantee that the final surface does not intersect itself, creates an excessive number of self-intersecting candidate solutions that must be discarded.

- **Numerical stability:** the method relies on floating-point arithmetic to constrain the topology of the final surface, a formulation in which rounding errors can be problematic. More specifically, the topology is constrained onto the sphere by detecting edge-edge intersections during the retessellation process. By the inherent nature of the spherical mapping, which aims to minimize regions with negative areas (i.e. negative Jacobian), topological defects correspond to extremely dense regions, with vertices potentially being as close as $10^{-5}$ mm. At this scale, floating-point rounding errors can occur and bring about “catastrophic” results: a non-detected intersection engendering a topologically-inconsistent retessellation with an incorrect Euler-number.

In this paper we propose a different approach that improves on our previous work. Specifically, we continue to identify topological defects as locally non-invertible regions, but no longer use the spherical mapping to produce candidate solutions. Instead, we generate potential solutions using the concept of non-separating loops and opening operators $[39, 52]$. Using this concept, the space of potential solutions is no longer restricted by the spherical mapping, resulting in a significantly more diversified set of candidate corrections, spanning a broader space. In addition, we avoid using floating-point arithmetic by working directly on the graph of the triangulation, thereby using exact arithmetic computations. Our method proceeds as follow:

1. Generate a mapping from the original cortical surface onto the sphere that is as close as possible to a homeomorphism (which was termed a maximally-homeomorphic mapping or a quasi-homeomorphic mapping in [42] and in [44]). Each topological defect is then identified as a set of overlapping triangles on the sphere.

2. For each topological defect, randomly generate sets of non-separating loops, and correct for the topology of the defect by opening and sealing the surface. The final maximum-a-posteriori configuration is selected in a Bayesian framework.

The method is a wholly self-contained topology correction algorithm, which determines geometrically-accurate and topologically-correct solutions based the MRI intensity profiles and the expected local curvature.
3 Identification of Topological Defects

We identify the presence of topological defects in the surface $C$ by computing its Euler-characteristic $\chi(C)$. In the presence of topological defects (i.e. $\chi \neq 2$), a mapping from the cortical surface $C$ onto the sphere $S$ that is homeomorphic over the majority of the manifold is generated, and we identify each defect as a set of overlapping faces. This step is identical to the approach developed by Fischl et al. in [42] and used in [44].

Briefly, identifying topological defects begins with the inflation and projection of the cortical surface $C$ onto a sphere $S$. The next step is to create a mapping $\mathcal{M} : C \rightarrow S$ by minimizing an energy functional $E_{\mathcal{M}}$ that directly penalizes regions in which the determinant of the Jacobian matrix of $\mathcal{M}$ becomes zero or negative; these regions are non-homeomorphic regions (i.e. locally non-invertible). The final step is to identify the topological defects by regions, where the homeomorphism is broken (i.e. regions with negative determinant or, equivalently, regions with overlapping faces). To clarify, the following steps in detail are taken from [42].

3.1 Initialization of the Mapping: Spherical Inflation

The initial mapping of the cortical surface to that of a sphere can be made by simply projecting each point of the cortical surface to the closest point on the sphere. In so doing, large regions of the initial mapping become non-homeomorphic.

In contrast, we use a simple procedure to unfold and smooth the folded cortical surface such that it approaches the surface of a sphere whose origin is the centroid of the initial surface. The algorithm consists of iteratively updating the position of each vertex based on a smoothness force $F_S$, and a radial spherical force $F_R$:

$$x_k(t + 1) = x_k(t) + F_S(t) + \lambda R F_R(t)$$

where $x_k$ is the position of the $k^{th}$ vertex at iteration number $t$ and the smoothness force $F_S$ is given by:

$$F_S = \frac{1}{N_k} \sum_{j \in N_k} (x_j - x_k) - \frac{1}{V} \sum_{i=1}^{V} \sum_{j \in N_i} (n_i n_i') . (x_j - x_i)$$

where:
- $N_k$ is the set of vertices neighboring the $k^{th}$ vertex;

---

6The Euler number of a surface is a topological invariant (For a tessellation, it can be easily computed as: $\chi = \#vertices - \#edges + \#faces$).
7$\chi = 2$ does not ensure the accuracy of the initial surface $C$, but this problem is out of the scope of this paper.
- $V$ is the number of vertices in the tessellation;
- $n_k$ and $n'_k$ are the surface normals at location $k$ and its transpose, respectively.

The smoothness term $F_S$ moves each vertex in the direction of the centroid of its neighbors, while projecting out the average inward movement created over the entire surface. The radial term simply drives each vertex toward the surface of a sphere with the desired radius $R$:

$$F_R = (R_k - x_k)$$

3.2 Quasi-Homeomorphic Mapping

Once the initial spherical configuration has been established, we generate a mapping $M$ that is as close as possible to a homeomorphism. A mapping is a homeomorphism if the determinant of its Jacobian matrix is non-singular, and the mapping itself is continuous. This is of course the multidimensional analog of monotonicity. In creating the mapping $M$, only its topological properties are concerned. To construct the mapping, we minimize an energy functional that directly penalizes regions in which the determinant becomes zero or negative, thus encouraging positive definiteness. Note that this is the only term in the energy functional - no preservation of metric properties is needed.

3.2.1 The Energy Functional

More specifically, noting that the Jacobian yields a measure of the deformation of an oriented area element under the mapping $M$, the energy functional $E_M$ constrains the penalization of compression primarily to negative semi-definite regions. If the initial area on the folded surface of the $i^{th}$ face is $A_i^0$, and the area on the spherical surface $S$ at time $t$ of the numerical integration is $A_i^t$, then the energy functional is given by:

$$E_M = \sum_{i=0}^{F} \frac{1}{k} \log (1 + e^{kR_i}) - R_i \text{ with } R_i = \frac{A_i^t}{A_i^0},$$

$$\frac{\partial E_M}{\partial A_i^t} = -\frac{1}{A_i^t(1 + e^{kR_i})}$$
The logarithmic nonlinearity restricts the penalization of compression primarily to negative semi-definite regions, as can be seen in the plot in Fig. 4a. \( R_i \) is an approximation of the Jacobian of the transformation \( \mathcal{M} \), \( R_i \approx J_{\mathcal{M}} = |\frac{\partial \mathcal{M}}{\partial \mathbf{x}}| \). The extent to which highly-compressed positive definite regions are penalized is determined by \( k \). In practice, we used a value for \( k \) of 100.

![Figure 4: a) Non-linearity of the energy functional \( E_M \) b) Triangle properties.](image)

### 3.2.2 Numerical Implementation

In order to complete the definition of the topology term of the energy functional, we consider the \( i^{th} \) triangle in the surface tessellation, with edges \( a_i \) and \( b_i \) connecting the vertex \( x_i \) to two of its neighbors \( x_l \) and \( x_j \) respectively. In the spherical representation, the normal vector field can be given a consistent orientation on the surface using the embedding space, and \( A_i \) becomes an oriented area, which may take on negative values - indicating folds in the surface. The normal vector is chosen to be pointing outward on the surface of the sphere \( \mathbf{n}_i = \frac{x_i}{||x_i||} \) (the sphere is centered at the origin).

Using the chain rule, the directional derivative of \( E_M \) with respect to the position of the \( k^{th} \) vertex:

\[
\frac{\partial E_M}{\partial x_k} = \frac{\partial E_M}{\partial A_i^t} \frac{\partial A_i^t}{\partial x_k} \quad (6)
\]

The first factor is given by Eq. [5]. The second is the change in the area of the \( i^{th} \) triangle caused by moving the \( k^{th} \) vertex, which can be computed from the prior description of the metric properties of the tessellation using the chain rule:

\[
\frac{\partial A_i^t}{\partial x_k} = \frac{\partial A_i^t}{\partial x_l} \frac{\partial x_l}{\partial x_k} + \frac{\partial A_i^t}{\partial x_j} \frac{\partial x_j}{\partial x_k} \quad (7)
\]

\(^8\)This is always possible except in pathological cases, such as the Mobius strip, that are non-orientable.
with
\[
\frac{\partial A_i}{\partial a_i} = b_i \land n_i, \quad \frac{\partial A_i}{\partial b_i} = n_i \land a_i.
\]  
(8)

The partials of the change in the legs with respect to a change in the vertex position are dependent on what position the vertex in question occupies in a given triangle:

\[
\frac{\partial a_i}{\partial x_k} = \begin{cases} 
[-1, -1, -1]^T & k = i \\
[1, 1, 1]^T & k = l \\
[0, 0, 0]^T & \text{otherwise}
\end{cases}
\]  
(9)

\[
\frac{\partial b_i}{\partial x_k} = \begin{cases} 
[-1, -1, -1]^T & k = i \\
[1, 1, 1]^T & k = j \\
[0, 0, 0]^T & \text{otherwise}
\end{cases}
\]  
(10)

### 3.3 Identification of Topological Defects

The resulting mapping \( M \) - from the initial tessellation \( C \) to the sphere \( S \) - is homeomorphic over the majority of the manifold. The surface is then examined for regions of non-invertibility, since these are the areas in which the current tessellation must be corrected to ensure proper topology. Multivalued regions, containing overlapping triangles\(^9\), constitute topological defects where the mapping is non-homeomorphic (these regions contain non-contractible curves, i.e. handles). \( M \) associates at each vertex \( v \) of the initial cortical surface \( C \) a vertex \( v_S = M(v) \) on the sphere \( S \). Vertices with spherical coordinates that intersect a set of overlapping triangles are marked as defective, with topological defects identified as connected sets of defective vertices.

Once a defect \( D \) has been identified, we compute its number of topological defects (i.e. handles). This number \( g(D) \) is called the genus and is related to the Euler number of the defect \( \chi(D) \) by the formula: \( g(D) = \frac{1 - \chi(D)}{2} \). An Euler number \( \chi(D) = 1 \) implies that the topology of the defect\(^10\) is the one of a disk (i.e. a patch without any handles), and the defect can be discarded from the list. Note that defects having an Euler characteristic \( \chi(D) = 1 \) indicates that the spherical mapping (sec. 3.2) is not precisely maximally-homeomorphic and that the minimization of the energy functional \( E_M \) has reached a local minimum. However, while the method might mistakenly identify some topologically-planar patches of surfaces that are subsequently discarded from the list, all topological defects are discovered. In our experience, the false positive rate of the spherical mapping is quite low (less than 10% of the defects are mistakenly identified).

\(^9\)In practice, we detect topological defects as sets of self-intersecting edges.

\(^{10}\)For a topological defect \( D \) with \( n_f \) faces, \( n_e \) edges, and \( n_v \) vertices (out of which \( n_b \) are border vertices), we always have the relations: \( n_v - n_e + n_f = \chi(D) \) and \( n_e = \frac{3n_f}{2} + \frac{n_b}{2} \).
We note that other methods could have been used to locate the topological defects in the manifold. For instance, the choice of the energy used to generate the spherical mapping is not unique. The energy $E_M$ defined above minimizes the area of locally non-invertible regions, generating extremely dense regions with vertices potentially being as close as $10^{-5}\text{mm}$. Previous methods [42, 44], which are quadratic in the number of defective vertices, suffers from large computational time for reasonably large defects. This motivates the use of an energy that pushes the topological defects into minimal non-homeomorphic regions. However, one could have generated a different spherical mapping by using a simple relaxation procedure to iteratively move each vertex toward the spherical average of its neighboring vertex positions [53, 54, 55].

3.3.1 Numerical Implementation

Once the spherical mapping has been generated, we detect the topological defects as connected sets of self-intersecting edges. Using a spatial look-up table, we first mark all self-intersecting edges (see [19] pp117) and subsequently segment them into connected components. As such, each connected component is not guaranteed to form a “valid” topological defect, i.e. a simply-connected region separated from the rest of the manifold by a unique border. The minimization of the energy $E_M$ which pushes defective vertices into extremely dense regions does not guarantee the simple-connectedness of the self-intersecting edge components. In addition, two overlapping faces do not always produce self-intersecting edges (think about one face being completely inside another one - the edges of the first triangle do not intersect the edges of the other triangle - however, note that the edges of the second triangle will necessarily intersect some other edges since the first triangle must be connected to the rest of the triangulation).

In order to correctly identify the topological defects as “valid” regions, we iteratively “augment” each connected set of self-intersecting edges by adding non-self-intersecting edges that are not connected to the largest connected region of non-self-intersecting edges. We segment the non-self-intersecting edges into connected regions and only keep the largest one (the “quasi-homeomorphic” region). Other regions are added to their neighboring topological defects. By doing so, we ensure that each defect forms a “valid” topological defect.

4 Geometrically-Accurate Topology Correction

Given a topological defect, our goal is to “optimally” correct its topology, that is, to generate a defect as geometrically-accurate and topologically-correct as possi-

\footnote{We consider the edge graph, in which two edges are connected if they share a vertex.}
Figure 5: a) Identification of two associated non-separating loops on a topological defect containing only one single handle. b-f) Topological correction using the “red” non-separating loop. Opening (b), sealing (c), and smoothing (relaxation procedure) of the two attached pre-tessellated disks (c-f). This correction amounts to filling the hole of the defect. g-l) Topological correction using the “purple” non-separating loop. This correction corresponds to cutting the handle of the defect. The “purple” loop, which is the shortest one in the defect, is the first selected one.

ble, given the available information. Correcting the topology amounts to finding the handles present in the defect and removing them. In algebraic topology, a handle (or, equivalently, a hole - Fig. 2h) in a 2-D smooth manifold is indicated by the presence of a simple closed curve that cannot be continuously deformed on the manifold into a single point. These non-contractible curves are called non-separating loops, since they do not partition the space (i.e. the manifold) into two connected components. This concept is closely related to Morse functions, Reeb graph theory, and homotopy [45, 46, 47]. The removal of non-separating loops reduces the genus of a manifold by cutting/opening the surface along the closed curves; they have been extensively used in graphics [39, 56, 57], combinatorial and computational topology [58, 52, 59, 60, 61], and medical imaging [40].

Our approach is based on the concept of non-separating loops: we correct the topology of each defect by opening and sealing the surface along an array of non-separating loops, assessing the resulting surface for optimality with respect to a Bayesian energy functional. This method is similar to (and was inspired by) the approach proposed by Guskov and Wood in [39]. Once a non-separating loop has been identified, we simply discard its faces and close the open mesh by attaching two pre-tessellated disks. We randomly generate a set of non-separating loops and select the maximum-a-posteriori correction in a Bayesian framework, monitoring the accuracy as well as the validity of the solution. For each of the

12We ensure that the final solution has the correct topology and does not self-intersect.
candidate solutions, an active contour optimization enhances the accuracy of the topologically-corrected surface.

**Algorithm 1 Geometrically-Accurate Topology Correction**

\begin{verbatim}
for all Defect \( \mathcal{D} \) such that \( \chi(\mathcal{D}) \neq 1 \) do
    for \( 1 \leq n \leq g(\mathcal{D}) \) do
        for \( 1 \leq i \leq \text{numbers of attempts} \) do
            Random generation of a non-separating loop \( \mathcal{L}_i \),
            Mesh opening by discarding the loop faces
            \( \mathcal{D}_i = \mathcal{D} \setminus \mathcal{L}_i \),
            Sealing of the mesh \( \mathcal{D}_i \),
            Smoothing and self-intersection test.
            Active Contour Optimization \( \mathcal{D}_i \).
        end for
    \end{verbatim}

\( \mathcal{D} = \arg\max \ p(\mathcal{D}_i | \mathcal{C}, I) \)

end for
end for

Our method is able to produce topological corrections corresponding to cutting the handle or filling the associated hole, depending on the identified non-separating loop. Moreover, note that filling the associated hole is equivalent to cutting the background handle identified by a non-separating loop. Figure 5 illustrates this point by showing two non-separating loops that result in different topological corrections. In addition, the position of the loop (the exact path onto the triangulation) determines the shape of the final corrected surface.

### 4.1 Generation of Non-Separating Loops

In the discrete formulation a non-separating loop is a set of connected faces that does not divide the rest of the triangulation into two connected components\(^{13}\). The concept of non-separating loops on a tessellation was introduced in the graphics community for topological noise removal by Guskov and Wood [39]. In their work, a selected vertex is used to initialize a region growing algorithm, which detects loops (i.e. topological defects) in the triangulation where wavefronts meet. Topological corrections are then obtained through the use of opening and sealing operators on the triangle mesh. While their method is fast, it depends on the initially selected vertex and does not guarantee a valid geometrically-accurate surface, as such corrections may create self-intersections. In addition, their approach,\(^{13}\)

\[^{13}\text{For a non-separating loop } \mathcal{L} \text{ on a manifold } \mathcal{C}, \chi(\mathcal{L}) = 0 \text{ and } \chi(\mathcal{C}) = \chi(\mathcal{C} \setminus \mathcal{L}).\]
as well as \cite{40,52}, is limited to triangulations produced by a fast-marching algorithm \cite{62,63}, while requiring a closed surface.

In contrast, the method we propose is not constrained to specific types of triangulations, nor does it necessitate a closed surface. The only requirement is that the initial surface be a valid triangulation, i.e. one for which each vertex possesses only one ring of connected neighbors\footnote{Marching cube algorithms are often used to produce triangulations from 3D images (distance functions or binary segmentations); recent marching cube algorithms \cite{63,28} generate valid triangulations without consistency problems. In practice, the initial cortical surface is produced from a binary digital volume by tessellating every square face of border voxels into 2 triangles\cite{3}. In order to guarantee that the resulting surface is a valid triangulation, one has to ensure that the digital segmentation has no critical configurations (i.e. “edge-edge” or “corner-corner” configurations), which is achieved by iteratively adding small sets of binary voxels to critical configurations.}. Similar to the approach of Guskov and Wood, we use a local wave front to locate handles in a defect \( D \). We evolve a front of triangles on the triangulation until we detect a front intersection. Caution is warranted in dealing with an open surface (see Fig.\,\cite{7}). Given a randomly selected seed face \( f_s \), we evolve a front of faces by fast-marching on the triangulation using approximated geodesic distances\footnote{We approximate the distance between two triangles by the Euclidean distance in between their center of gravity.}. At each step, we check if the candidate face induces a front intersection, detected as the intersection of two triangles, \( f_a \) and \( f_b \), in a single vertex (Fig.\,\cite{7b}-d). Once a front intersection has been detected, we examine if the remaining faces in the triangulation are connected. In this case, we have identified a non-separating loop in the defect. Additionally, when the remaining faces are connected inside the defect \( D \), a second non-separating loop is identified (Fig.\,\cite{7b}), and we randomly select one of them. Otherwise, we resume the front evolution (Fig.\,\cite{7d}). We note that an efficient implementation of the fast-marching method is attainable using the min-heap data structure, resulting in a complexity of \( O(n_f \log n_f) \), where \( n_f \) is the number of faces in the defect. The identification of a connected path in the remaining faces of the defect can also be achieved by fast-marching on the triangulation, starting from two neighboring faces of \( f_a \), or \( f_b \), since \( f_a \) and \( f_b \) intersects in a single vertex (magnified region of Fig.\,\cite{7b}).

Once a valid wavefront intersection has been detected, we extract a non-separating loop (i.e. a set of connected faces) by back-tracking the faces starting from the front intersection (i.e. \( f_a \) and \( f_b \)) until we reach the initial seed face \( f_s \). We “close” the loop at the front intersection (since \( f_a \) and \( f_b \) intersects in a single vertex - Fig.\,\cite{7b}) by adding the shortest path of defect faces that connects \( f_a \) to \( f_b \). Details on the implementation are presented in the next section.
4.1.1 Numerical Implementation

Once a wavefront intersection has been detected, a couple of careful steps need to be taken in order to extract a valid non-separating loop. A valid non-separating loop is a set of connected faces that does not divide the rest of the triangulation into two connected components. Also, every face of a non-separating loop should be connected to exactly 2 other faces of the loop.

Back tracking of the faces: we first back-track the faces starting from the front intersection (i.e. \( f_a \) and \( f_b \)) until the two disconnected back-tracking paths intersect in a common vertex. Back-tracking the faces until the initial seed face \( f_s \) is reached would probably produce a non-valid loop, one for which some faces are connected to more or less than 2 loop faces. Once an intersection has be found, one simply need to close the loop at its two ends (the newly-identified intersection and the front intersection).

![Figure 6: Cutting of the non-separating loop faces into distinct faces and opening of the surface.](image)

Closing the loop: at this point, we simply need to “close” the loop at the two intersections to obtain a valid non-separating loop. We generate the final non-separating loop by adding the shortest path of defect faces that connects each side of the intersection. In the case of the front intersection, this means adding the shortest path that connects \( f_a \) to \( f_b \). In doing so, one has be to careful not to add new connections to the set of faces (the final loop is composed of faces that are connected to exactly 2 other faces).

Cutting the loop faces in 3 distinct faces: finally, after a valid non-separating path has been found, we divide every loop triangle into 3 triangles. To do so, we add two vertices to the two triangle edges that belong to the non-separating loop (the remaining edge is part of the boundary of the loop) and connect them by an edge: this newly-added edge will be the location of the surface cut in the next section.
4.2 Reducing the Genus: Cutting and Sealing the Open Surface

4.2.1 Sealing the Cut

Once a non-separating loop has been found, we discard the faces of the loop and seal the surface by attaching two pre-tessellated disks (i.e. patches without any topological defects) to both open sides of the defect. The attachment procedure is a graph operation designed to find a set of connecting edges and corresponding faces between two rings of vertices formed by one open side of the defect and the border of the disk. The exact shape of the surface and the pre-tessellated disk (i.e. the locations of its vertices) is not important in the attachment of the two disks.

To attach a pre-tessellated disk to one open side of the defect (Fig. 8b-d), we assume that the border vertices of the disk of the curve are regularly spaced along a unit disk $U(0, 1)$. We also assume that the vertices of the open side of the defect
are spaced along a disk \( \mathbb{U}(0, 2) \) proportionally to their geodesic distance on the surface. Attaching the disk to the surface amounts to finding a triangulation from one to the other. In practice, we use a Delaunay triangulation.

For visualization purposes, we have set the position of each attached vertex to the average of the positions of the vertices constituting the loop (Fig. 5c-h and Fig. 8b-d): the vertices of each pre-tessellated patch have, after attachment, the same spatial location. The size of the disk used to patch the surface is based on the size of the contour or the defect, with larger disks used to seal larger patches. Fig. 8a shows some typical patches.

Figure 8: a) Examples of pre-tessellated disks used in the sealing process. b) View of an open surface before attachment. c-d) Attachment operation. c) The pre-tessellated disk (in gray) has 5 vertices, regularly spaced along a unit disk \( \mathbb{U}(0, 1) \), and 4 faces. The yellow triangles are the ones that are necessary to attach the disk to the open surface. To this end, the defect vertices are spaced along \( \mathbb{U}(0, 2) \) proportionally to their geodesic distance on the surface and a Delaunay triangulation is generated. d) The position of the newly attached vertices (i.e. the 5 vertices of the pre-tessellated disk) is first set to the average of the positions of the vertices constituting the loop. e) View of the sealed surface after a relaxation procedure on the disk vertices.

### 4.2.2 Smoothing and Self-Intersection Check

The attachment operation reduces the genus of the surface by one. Although the intrinsic topology has been modified, we have not yet produced an accurate correction since all attached vertices have the same inaccurate location (the average of the positions of the loop vertices - Fig. 8d). Our goal is to optimize the shape of the surface by modifying the position of the defect vertices (particularly the position of the newly-attached vertices) in order to maximize the goodness of fit of the corrected defect. We also need to ensure that no self-intersections are created by topological corrections. In mathematical terms, we can express this condition as:

\[
D_{\text{final}} = \arg\max_{\text{valid } D_i} g(D_i, I, C),
\]

where \( g(D_i, I, C) \) is a measure of the goodness of fit of the candidate solution \( D_i \), and “valid” \( D_i \) designates a non-self-intersecting patch. This continuous formu-
lation of the problem does not translate easily into a discrete framework, in which self-intersections are difficult to detect and prevent.

For this reason, we take a somewhat different approach. After attachment, we iteratively update the positions of the newly-added vertices by iteratively averaging their location with that of their neighbors (we also include in the relaxation procedure the two first neighbors of the newly-attached vertices). This process is illustrated by Fig. 5. After convergence, we check if the sealed surface self-intersects and discard non-separating loops that lead to self-intersecting surfaces (Fig. 9b,c). We detect self-intersecting faces using a discretized (1-mm voxels) spatial look-up table to constrain the self-intersection check to faces in the local neighborhood that have been updated in the relaxation process.

Figure 9: a) A complex topological defect with 3 handles. b) A large non-separating loop that leads to a self-intersecting patch (c). Such non-separating loops are detected and discarded from the topology correction process. d) Geometrically-accurate topological correction.

The relaxation procedure is quite fast (a hundred iterations sufficient to achieve convergence), and most of the time yields a valid solution, i.e. a patch that does not self-intersect. In our experience, less than 5% of the patches self-intersect. Figure 10 provides some examples of valid candidate patches that have had their genus reduced by one (i.e. candidate patches with one handle less than the initial patch). Note the wide range of potential solutions. Finally, once a valid topological correction is obtained, we improve the accuracy of the correction by maximizing a fitness function $g(D, I, C)$ defined in the next section, using an active contour formulation.

### 4.3 Fitness and Likelihood Functions

Many valid topological corrections exist (Fig. 10), and one would like to select the solution that maximizes both the goodness of fit of the final surface given the available image information, as well as one that conforms to our prior knowledge about the cortex. A cortical surface is a smooth manifold $C$ that partitions the embedding space into an interior, composed of white matter (as well as deep gray
structures and ventricles far from the surface), and an exterior, composed of gray matter. We characterize the goodness of a retessellation by measuring two of its properties:

1. The smoothness of the resulting surface,
2. the MRI values $I$ inside and outside the surface.

We define the fitness function as the posterior probability of observing the corrected defect $D_i$ given the MRI intensity values and the surface $C$: $g(D_i, I, S) = p(D_i | C, I)$. Formally, the posterior probability of the $i^{th}$ retessellation $D_i$ is given by:

$$p(D_i | C, I) \propto p(I | C, D_i) p(D_i | C).$$

The likelihood term $p(I | C, D_i)$ encodes information about the MRI intensities inside and outside the surface. Each corrected defect separates the underlying MRI volume into two distinct components, an inside part $C^-$ and an outside part $C^+$. An acceptable candidate solution should create a space partition with the majority of its inside and outside voxels corresponding to white and gray matter voxels, respectively. In order to estimate the likelihood $p(I | C, D_i)$, we assume that the noise is spatially independent. This probability can then be rewritten as:

$$p(I | C, D_i) = \prod_{x \in C^-} p_w(I(x) | C, D_i) \prod_{x \in C^+} p_g(I(x) | C, D_i) \; \text{volume-based information} \quad \prod_{v=1}^{V_i} p(g_i(v), w_i(v) | C, D_i) \; \text{surface-based information}.$$  

\[16\] We use the angle-weighted pseudo-normal algorithm [63] to compute the signed distance of the tessellation. The voxel grid is partitioned into inside negative values and outside positive values.
\[ p_w(I(x)|C, D_i) \] and \[ p_g(I(x)|C, D_i) \] are the likelihood of intensity values at location \( x \) in the volume inside and outside the tessellation respectively, \( p(g_i(v), w_i(v)|C, D_i) \) is the joint likelihood of intensity values inside and outside the tessellation at vertex \( v \) in tessellation \( D_i \).

Geometric information can be incorporated via \( p(D_i|C) \), which represents priors on the possible retessellation. For example, \( p(D_i|C) \) could have the form:

\[
p(D_i|C) = \prod_{v=1}^{V_i} p(\kappa_1(v), \kappa_2(v)|C),
\]

where \( \kappa_1 \) and \( \kappa_2 \) are the two principal curvatures of the surface, computed at vertex \( v \). In our experience, this choice of priors produces accurate candidate solutions.

Figure 11: a) Examples of the gray and white matter distributions estimated locally from a given a topological defect. b) Joint distribution of gray and white matter given the surface computed, using the non-defective portion of the gray/white boundary representation of a single subject. The gray and white matter intensity are two correlated variables, as indicated by the diagonal structure of the joint distribution. c) Joint distribution of two principal curvatures of the surface.

Given that the vast majority of the surface is in general non-defective, we have an ample amount of data to estimate the correct forms of the distributions \( p(D_i|C) \), \( p_g(I(x)|C, D_i) \), \( p_w(I(x)|C, D_i) \) and \( p(g_i, w_i|C, D_i) \). In particular, the single tissue distributions \( p_g(I(x)|C, D_i) \) and \( p_w(I(x)|C, D_i) \) are estimated locally around each topological defect, in a region that excludes the defect itself (the region is taken as a cubic box containing the defect \( \pm 5 \text{mm} \)). This makes the procedure completely adaptive and self-contained, in the sense that no assumptions need to be made about the contrast of the underlying MRI image(s), and no training or parametric forms are required for \( p(D_i|C) \). An example of the estimation of \( p(g_i, w_i|C, D_i) \) and \( p(D_i|C) \) is given in Fig. 11. Image b) shows the joint distribution of gray and white matter given the surface computed, using the non-defective portion of the
gray/white boundary representation of a single subject. Note the diagonal character of the distribution that indicates the mutual dependence of the intensities - brighter white matter typically means brighter gray matter due to factors such as bias fields induced by RF inhomogeneities and coil sensitivity profiles, as well as intrinsic tissue variability. One possible form of the priors on the tessellation is given by Fig. 11c, which shows the joint distribution of the two principal curvatures $\kappa_1$ (green) and $\kappa_2$ (red) computed over the non-defective portion of a single surface.

### 4.4 Optimization Using Active Contour Patches

During the search, candidate patches $\mathcal{D}_i$ are selected based on their fitness value $p(\mathcal{D}_i|\mathcal{C}, I)$. After attachment, smoothing, and the self-intersection check, each patch defines a valid manifold that can be treated as an active contour with fixed boundaries. Each patch is locally deformed in order to maximize the posterior probability $p(\mathcal{D}_i|\mathcal{C}, I)$. Instead of deriving the exact Euler-Lagrange equation of the active contour $\mathcal{D}_i$ for the energy functional $p(\mathcal{D}_i|\mathcal{C}, I)$, we use an approximation procedure. We note that the fitness function of a candidate solution measures the smoothness of the resulting surface and the MRI intensity profile inside and outside the surface. We simply update the position of each interior vertex $x_k$ of the candidate tessellation based on a smoothness force $F_S$ and an MRI intensity-based force $F_M$:

$$x_k(t + 1) = x_k(t) + F_S(t) + \lambda_M F_M(t).$$  \[12\]
The smoothness force is the same as the one defined in Eq. 2. The intrinsic curvature-based force enforces a smoothness constraint on the deformed active contours and tends to maximize the prior term $p(D_i|C)$. The MRI intensity-based force $F_M$ is designed to drive the active contour towards the true boundary separating the gray from the white matter:

$$F_M = [T_v - I(x_k)]\nabla I(x_k),$$

where the targeted value $T_v$ is computed from the gray and white matter distributions. The mean intensity and variance of the gray and white matter intensities are estimated from the respective distributions $p_g$ and $p_w$, denoted by $\mu_g$, $\sigma_g$, $\mu_w$, and $\sigma_w$, and the local threshold $T_v$ is computed using the following equation:

$$T_v = \frac{\mu_w\sigma_g + \mu_g\sigma_w}{\sigma_w + \sigma_g}. \quad (14)$$

At each iteration, we measure the exact fitness function $p(D_i|C, I)$ of the active contour and stop the deformation when the fitness function is maximized. We ensure that the surface remains a valid one by preventing self-intersections during the active contour optimization. The constant $\lambda_M$ is empirically set to 0.5.

### 4.5 Implementation Parameters

The proposed approach is implemented using the following parameters. A typical topological defect contains on the order of 100 faces. For a defect containing $n_f$ faces, we produce $n_f/3$ patches per handle by generating non-separating loops from “semi-randomly” selected seed faces. We use spatial information to better distribute randomly selected faces inside the defect by ensuring that a seed face is not drawn twice and that a new draw is not neighboring the previous one. This procedure produces random selection of seed faces that are likely to cover the defect uniformly. As such, our method samples likely solutions with high probability. In addition, the first non-separating loop is always the smallest loop in the defect. To identify the smallest loop, we simply generate as many loops as there are faces in the defect and select the shortest one. Note that the loop generation is computationally fast (in $O(n_f \log n_f)$), while detecting self-intersections and the computation of the fitness function are the slowing factors.

### 5 Validation

We have applied our proposed approach to 43 real images (this data set is the same one used for the evaluation of our previous method [44]). The dataset is comprised
Figure 13: a) Original defect: red and green vertices represent inside and border vertices respectively. b) One sagittal view of the defect. c) Original defect. The vertices in the circled regions have by the spherical mapping the same location on the sphere. d) Incorrect solution generated by the genetic algorithm using the spherical mapping. This solution corresponds to the best candidate within the space of potential retessellation constrained by the initial spherical mapping. e) Solution generated by our approach. One can check the similarity of our solution with the one generated by a trained operator in Fig. 3.

Validation data came from several data sets. They were a mix of pulse sequence (SPGR, MP-RAGE), scanner types (Siemens 1.5T, GE 1.5T) and pathology (normal control, schizophrenic and Alzheimer’s).

Seventeen scans were acquired in 2000/2001 using a Siemens Sonata system with the following parameters: TR: 7.25 ms; TE: 3.22 ms; TI: 600.00 ms; flip angle: $7.00^\circ$; 1.3-mm sections (resampled to 1 mm isotropic). This data set consists of 8 young (YNC), 7 elderly normal controls (ENC), and 2 Alzheimer’s (AD).

The second data set was acquired using a Siemens Vision system in 1994/1995 with the following parameters: TR: 9.70 ms; TE: 4.00 ms; TI: 621.00 ms; flip angle: $10.00^\circ$; 1.25 sections (resampled to 1 mm isotropic). Data comes from studies reported in Buckner et al. [65] and Logan et al. [66] and also later subjects imaged using the same anatomic protocol [17]. This data set consists of 6 Young Normal Control, 14 non-demented and 6 demented adults.

[17] We thank Randy Buckner and the Washington University Alzheimer’s Disease Research Center for providing the data set.
5.2 Discussion of the Results:

5.2.1 Accuracy

Methods that do not integrate statistical and geometric information will often fail to produce solutions comparable to those that a trained operator makes. This is illustrated in Fig. [12]a,b, where valid solutions do not always correspond to minimal corrections (i.e. cutting the handles in the magnified examples of Fig. [12]a,b). Only general approaches that integrate additional information can produce correct solutions.

In order to assess the accuracy of the proposed method, we rely on experts to evaluate the solutions. All solutions matched the ones produced by a trained expert. Then, we compare our results with the ones obtained in [44]. To our knowledge, these two methods are the only ones proposed to explore the space of potential solutions for selecting the best correction to a topological defect. In most cases, both approaches produced almost-identical, correct solutions that other methods failed to produce. However, for some complex topological defects, our approach outperformed the genetic-search approach. As we previously noted, the space of potential retessellation explored by the genetic algorithm depends strongly on the the initial spherical mapping. For some large and complex topological defects, the genetic search does not develop correct solutions (i.e. the ones that a trained operator produces). The current approach, which does not rely on the spherical mapping to obtain candidate solutions, is more likely to generate the correct ones, as Fig. [13] illustrates.

Finally, to evaluate the quality of the corrections, we compute for each defect the average Hausdorff distance between automatically-corrected surfaces (using our method) and manually-corrected surfaces produced by a trained operator. The average Hausdorff distance is less than 0.2mm (similar to the results obtained in our previous method [44]).

5.2.2 Numerical implementation and computational time

An average cortical surface contains on the order of 50 topological defects, most of which are relatively small: most defects contain less than 50 vertices (approximately 100 faces[18], and are corrected in a few seconds.

Larger defects, with more than 100 vertices, can take a few minutes, due to the self-intersection check and the computation of the fitness associated with each defect. However, the overall process is no longer quadratic in the number of vertices contained in the convex hull of each defect, as in our previous method [44], and a

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[18] For a topological defect with \(g\) handles and \(n_v\) vertices out of which \(n_b\) are border vertices, the number of faces is \(n_f = 2(n_v - 1 + 2g) - n_b\)
full brain is corrected in approximately 20 minutes on a 1-GHz Pentium IV machine. More importantly, we stress that the whole process could be parallelized, as each defect is independent of one another, thus providing the potential for two order of magnitude increase in the speed of the procedure with parallelization.

In addition, our approach produces few self-intersecting patches (less than 5% of all generated patches). Most of the computation time is taken up by the calculation of the fitness and the self-intersection test, with a complexity approximately proportional to the size of the defect.

Finally, we emphasize that all of the topological operations (i.e. operations that lead to the correction of the topology) are exact arithmetic operations that manipulate the graph of the triangulation. In contrast to our previous genetic search approach that used the spherical mapping to constrain the topology of the final solution, and as such was sensitive to floating-point rounding errors, the current method always produces a valid final surface, i.e. one with the Euler-characteristic of a sphere $\chi = 2$.

5.2.3 Limitations

It is important to state the limitations of the proposed approach. While the current algorithm is no longer constrained by the spherical mapping, not all configurations can be attained. The obvious explanation is the local behavior of our topological corrections, which, having identified potential cuts, optimize the shape of the surface locally by using a gradient descent within the active contour framework. Therefore, our approach may not be able to produce geometrically-accurate solutions in the event of extremely noisy MRI images.

However, in our experience such configurations are quite unlikely. Topological inconsistencies are often the result of very few mislabeled ambiguous voxels during the segmentation process, and local topological corrections are usually sufficient to produce geometrically-accurate solutions. While aware of these limitations, we do not believe this to be a real restriction to the geometrically-accurate topology correction of cortical surfaces from standard structural MRI acquisitions.

6 Conclusion

We have proposed an automated method to accurately correct the topology of cortical representations. Our approach integrates statistical and geometric information in selecting the best correction for each defect. Non-separating loops locate

\[\chi = 2\]
handles present in the volume, and produce topologically-corrected candidate solutions by discarding the faces that form the loops and by sealing the open mesh. The accuracy of each candidate solution is then maximized by active contour optimization. Finally, randomly-generated candidate solutions are selected based on their goodness of fit in a Bayesian framework. The fitness of a retessellation is measured by the smoothness of the resulting surface and the local MRI intensity profiles inside and outside the surface. The resulting procedure is a wholly adaptative and self-contained topology correction algorithm, which determines geometrically-accurate, topologically-correct solutions based on the MRI intensity profiles and the expected local curvature.

The topology correction is fast, and a full brain can be corrected in about 20 minutes on a current (1-GHz Pentium IV) machine (approximately 40 minutes taking into account the generation of the spherical mapping). Exact arithmetic operations on the graph of the triangulation ensure that no floating-point rounding errors occur during the topology-correction process, at the same time guaranteeing that the final surface exhibits the topology of a sphere. To our knowledge, this approach has been the only one proposed thus far to explore the full space of potential solutions in order to select the best correction of a topological defect.

Finally, note that the proposed approach is not restricted to spherical topologies, and can be used to correct the planar topology of any triangulations.

This algorithm is part of the cortical surface reconstruction and flattening software Freesurfer, associated with \[3,5,6,10\]. Source code for the generation of non-separating loops, the opening and the sealing of topological defects is available at http://cermics.enpc.fr/~segonne/research/topologycorrection/.

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